Math 270 Day 6 Part 1

Section 2.5: Special Integrating Factors

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What we'll go over in this section

- What is a (general/special) integrating factor?

- Discussion/Formulas

- Solving a DE in differential form that is not exact using a special integrating factor

Section 2.5: Special Integrating Factors

First, a problem from section 2.4

Example 4 Show that

$$(x + 3x^3 \sin y)\, dx + (x^4 \cos y)\, dy = 0 \tag{16}$$

is *not* exact but that multiplying this equation by the factor x^{-1} yields an exact equation. Use this fact to solve (16).

Section 2.5: Special Integrating Factors

What is a (General/Special) Integrating Factor?

Integrating Factor

Definition 3. If the equation

$$M(x, y)\,dx + N(x, y)\,dy = 0 \tag{1}$$

is not exact, but the equation

$$\mu(x, y)M(x, y)\,dx + \mu(x, y)N(x, y)\,dy = 0\,, \tag{2}$$

which results from multiplying equation (1) by the function $\mu(x, y)$, *is* exact, then $\mu(x, y)$ is called an **integrating factor**[†] of the equation (1).

Section 2.5: Special Integrating Factors

What is a (General/Special) Integrating Factor?

Example 1 Show that $\mu(x, y) = xy^2$ is an integrating factor for $(2y - 6x)\,dx + (3x - 4x^2y^{-1})\,dy = 0$.
Use this integrating factor to solve the equation.

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Discussion/Formulas…How do we find the integrating factor $\mu(x, y)$?

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$$(1) \qquad M(x,y)\,dx + N(x,y)\,dy = 0$$

Special Integrating Factors

Theorem 3. If $(\partial M/\partial y - \partial N/\partial x)/N$ is continuous and depends only on x, then

$$(8) \qquad \mu(x) = \exp\left[\int \left(\frac{\partial M/\partial y - \partial N/\partial x}{N}\right) dx\right]$$

is an integrating factor for equation (1).

If $(\partial N/\partial x - \partial M/\partial y)/M$ is continuous and depends only on y, then

$$(9) \qquad \mu(y) = \exp\left[\int \left(\frac{\partial N/\partial x - \partial M/\partial y}{M}\right) dy\right]$$

is an integrating factor for equation (1).

Method for Finding Special Integrating Factors

If $M\,dx + N\,dy = 0$ is neither separable nor linear, compute $\partial M/\partial y$ and $\partial N/\partial x$. If $\partial M/\partial y = \partial N/\partial x$, then the equation is exact. If it is not exact, consider

$$(10) \qquad \frac{\partial M/\partial y - \partial N/\partial x}{N}.$$

If (10) is a function of just x, then an integrating factor is given by formula (8). If not, consider

$$(11) \qquad \frac{\partial N/\partial x - \partial M/\partial y}{M}.$$

If (11) is a function of just y, then an integrating factor is given by formula (9).

Section 2.5: Special Integrating Factors

Solving a DE in differential form that is not exact using a special integrating factor

Example 2 Solve $(2x^2 + y)\,dx + (x^2y - x)\,dy = 0$.